Modeling and Computational Method for Air-core Winding Based on the Overlapping Partitioning Waveform Relaxation

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In this paper, an elaborate broadband model for the air-core winding is created based on the partial element equivalent circuit (PEEC) approach in order to study the internal overvoltage under the lightning transients. Because the large scale of the resulting model makes it difficult to be solved directly, the waveform relaxation method combined with the overlapping partitioning techniques is used. In this method, the whole network is divided into many sub-circuit and solved separately based on the analysis of the winding's coupling parameters. Furthermore, the relationship of the convergence rate between the number of the overlapping areas is also discussed. In order to validate the method, an experiment air-core winding is studied, and a conclusion is achieved that the rate of the convergence can be improved greatly by using the overlapping partitioning techniques.

***Index Terms*—Computational modeling, Numerical analysis, Iterative methods, Convergence of numerical methods**

I. Introduction

Waveform relaxation (WR) method is an efficient method for the large scale circuit and has been succeeded in the simulation of the PCB interconnect system[1-5]. Recent studies has been found that vast steps are needed for waveform relaxation-transverse partitioning algorithm because of the strong coupling between the subsystems. To address this problem, a new WR method for multi-conductor system was proposed based on the idea of overlapping partitioning[3].

The winding is another typical structure widely used in the power system, which contains lots of conductors with strong coupling effect. In this paper, the model and solution method based on the WR is studied and a conclusion is achieved that the overlapping partitioning WR method is very suitable for the simulation of the winding type models.

II. PEEC-model of the Air-core Windings

Based on the PEEC approach, each turns of the winding can be viewed as a series lumped elements, and the coupling between them can be described by the partial inductance and partial capacitance. The equivalent circuit topology is shown in Fig. 1. Mutual capacitance and mutual inductance between the conductors are not shown but should be considered.

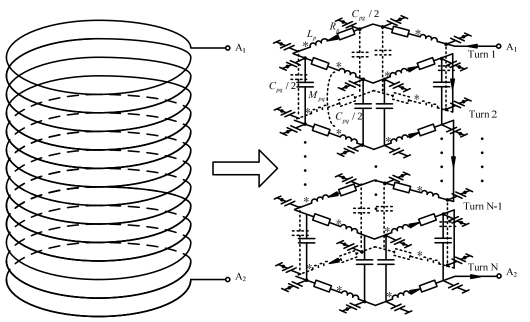

Fig.1 Structure of winding and its PEEC equivalent model

In order to analyze voltage distribution of winding, the circuit equations are specified in terms of the modified nodal analysis equations and shown as (1).

$$\underbrace{\begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{L} \end{bmatrix}}_{\boldsymbol{M}} \begin{bmatrix} \dfrac{d\boldsymbol{u}_n}{dt} \\ \dfrac{d\boldsymbol{i}_b}{dt} \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{0} & \boldsymbol{A} \\ \boldsymbol{A}^T & \boldsymbol{0} \end{bmatrix}}_{\boldsymbol{N}} \begin{bmatrix} \boldsymbol{u}_n \\ \boldsymbol{i}_b \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{i}_s \\ \boldsymbol{0} \end{bmatrix}}_{\boldsymbol{f}} \qquad (1)$$

where $\boldsymbol{C}$ represents nodal capacitance matrix, $\boldsymbol{L}$ represents branch inductance, $\boldsymbol{A}$ is node-branch incidence matrix, $\boldsymbol{u}_\text{n}$, $\boldsymbol{i}_\text{b}$ represent nodal voltage and branch current of inductance respectively, and $\boldsymbol{i}_\text{s}$ represents the current injecting to the node.

III. Overlapping Partitioning Waveform Relaxation Method

Because of thousands of turn numbers in the winding, the dimension of equations established is so large which is difficult to be calculated directly. In order to solve this problem, the system will be partitioned into several subsystems and solved by iteration based on the WR method.

As for the WR method, the rate of convergence depends on the natural coupling between subsystems. However, reactor winding wound tightly and the capacitive and inductive coupling is strong, traditional WR method cannot present coupling between the subsystems well, i.e. slow a bad convergence and low efficiency. In this paper, overlapping partitioning techniques is used, in which the whole winding is divided into several sub-circuit with some common conductors, which ensures the inherent strong coupling between sub-circuit. In this way, not only can retain the advantage of traditional WR algorithm of "divide and conquer", but also accelerate the convergence.

Based on the backward difference technique, (1) can be written as

$$\boldsymbol{P} \cdot \boldsymbol{X}^{n+1} = \boldsymbol{Q} \cdot \boldsymbol{X}^{n} + \boldsymbol{f}^{n+1} \qquad (2)$$

where $\boldsymbol{P} = (\dfrac{\boldsymbol{M}}{\Delta t} + \dfrac{\boldsymbol{N}}{2})$, $\boldsymbol{Q} = (\dfrac{\boldsymbol{M}}{\Delta t} - \dfrac{\boldsymbol{N}}{2})$.

In order to give a clear description, the system is divided into only 2 subsystems here. The matries $\boldsymbol{P}$ and $\boldsymbol{Q}$ can be denoted as

$$\begin{cases} \boldsymbol{P}=\begin{bmatrix}\boldsymbol{P}_{11} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{P}_{22}\end{bmatrix}-\begin{bmatrix}\boldsymbol{0} & \boldsymbol{P}_{12}\\ \boldsymbol{P}_{12}^{T} & 0\end{bmatrix} \\ \boldsymbol{Q}=\begin{bmatrix}\boldsymbol{Q}_{11} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{Q}_{22}\end{bmatrix}-\begin{bmatrix}\boldsymbol{0} & \boldsymbol{Q}_{12}\\ \boldsymbol{Q}_{12}^{T} & \boldsymbol{0}\end{bmatrix} \end{cases} \tag{3}$$

So the whole system can be divided into two independent sub-circuits, and each sub-circuit can be solved as

$$\begin{cases} \boldsymbol{X}_1^{n+1,r+1}=\boldsymbol{P}_{11}^{-1}\left(\boldsymbol{P}_{12}\boldsymbol{X}_2^{n+1,r}+\boldsymbol{Q}_{11}\boldsymbol{X}_1^{n}-\boldsymbol{Q}_{12}\boldsymbol{X}_2^{n}\right) \\ \boldsymbol{X}_2^{n+1,r+1}=\boldsymbol{P}_{22}^{-1}\left(\boldsymbol{P}_{21}\boldsymbol{X}_1^{n+1,r}+\boldsymbol{Q}_{22}\boldsymbol{X}_2^{n}-\boldsymbol{Q}_{21}\boldsymbol{X}_1^{n}\right) \end{cases} \tag{4}$$

where $\boldsymbol{X}_1$ and $\boldsymbol{X}_1$ are the node voltage and branch current vectors in network 1 and 2, respectively, n and r represent the nth moment and rth iteration, respectively. During the r+1th iteration at the n+1th moment the solution of the network can be obtained as

$$\boldsymbol{X}^{n+1,r+1}=\begin{bmatrix}\boldsymbol{E}_1\boldsymbol{X}_1^{n+1,r+1}\\ 0\end{bmatrix}+\begin{bmatrix}0\\ \boldsymbol{E}_2\boldsymbol{X}_2^{n+1,r+1}\end{bmatrix} \tag{5}$$

where $\boldsymbol{E}_1=\begin{bmatrix}\boldsymbol{1} & \\ & \boldsymbol{\alpha}\end{bmatrix}$, $\boldsymbol{E}_2=\begin{bmatrix}\boldsymbol{1}-\boldsymbol{\alpha} & \\ & \boldsymbol{1}\end{bmatrix}$ are weighting matrix[5], $\boldsymbol{\alpha}$ is a diagonal matrix that diagonal element less than 1, whose dimension is equal to common segment's.

IV. Example Analysis and Experimental Verification

An experimental winding with 100 turns is studied to validate the accuracy, which is shown as Fig. 2. The winding is composed by the aluminum wires covered with insulation whose relative dielectric constant is 3.2, the diameter of the wire and the diameter of winding are 3mm and 1m, respectively.

Fig. 2. Experimental schematic diagram.(① experimental winding, ②③ oscilloscope, ④ Pearson broadband current sensor shielded by box ⑤ surge generator)

A 1.2/50μs lighting wave is applied to the first end of the winding and the voltage at the 31st and 51st turns are measured, shown as Fig. 3. The simulation results are in good agreement with the measurement by comparison.

In order to illustrate the efficiency of the overlapping partitioning method, both of the convergence steps according to the traditional WR and overlapping partitioning WR are shown in Fig. 4, as can be seen, the convergence rate of the overlapping partitioning WR is much more faster than the traditional WR method.

V. Conclusion

In this paper, a time-domain WR method is proposed for the voltage distribution of winding type equipment problem, and the accuracy is verified by experiment. In order to improve the computational efficiency of the algorithm, an overlapping partitioning WR method is introduced. As a result, the overlapping partitioning WR proposed has achieve a better rate of convergence. In addition, realization of parallel computing is the follow-up work.

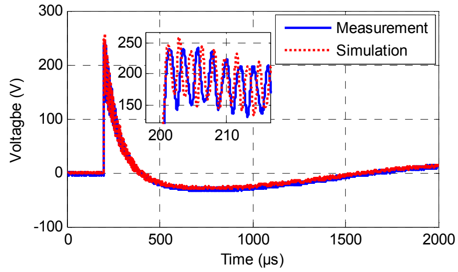

Fig. 3(a) Results of measurement and computation of 31st turn.

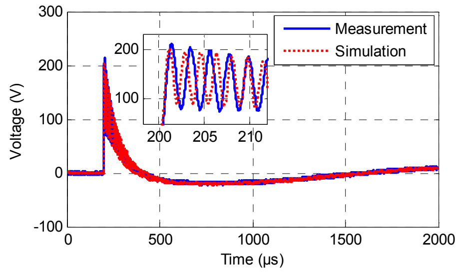

Fig. 3(b) Results of measurement and computation of 51st turn.

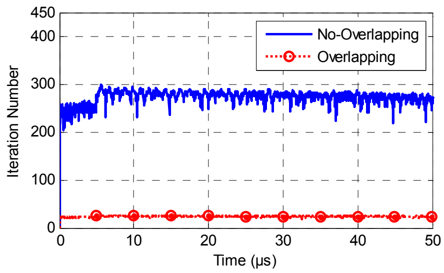

Fig. 4. Iterations using the method of traditional WR and overlapping partitioning WR.

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